

Hong Kong Mathematics Olympiad (1996 – 97)

Heat Event (Group)

香港数学竞赛 (1996 – 97)

初赛项目 (团体)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

1. If a_1, a_2, a_3, \dots and b_1, b_2, b_3, \dots are arithmetic sequences, where $a_1 = 25$, $b_1 = 75$ and $a_{100} + b_{100} = 100$. Find the sum of the first 100 terms of the sequence $a_1 + b_1, a_2 + b_2, \dots$

已知 a_1, a_2, a_3, \dots 和 b_1, b_2, b_3, \dots 为等差数列，其中 $a_1 = 25$, $b_1 = 75$ 及 $a_{100} + b_{100} = 100$ 。

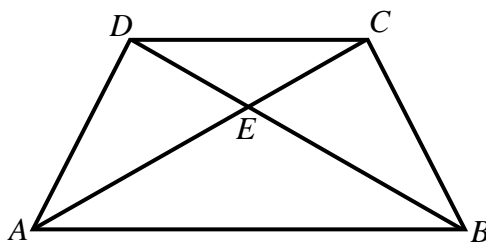
求数列 $a_1 + b_1, a_2 + b_2, \dots$ 的前 100 项的和。

2. If $f(x) = \frac{2x}{x+2}$ and $x_1 = 1$, $x_n = f(x_{n-1})$, find x_{99} .

已知 $f(x) = \frac{2x}{x+2}$ ，及 $x_1 = 1$, $x_n = f(x_{n-1})$ ，求 x_{99} 。

3. $ABCD$ is a trapezium, where $AB \parallel DC$ and area of $\triangle DCE$: area of $\triangle DCB = 1 : 3$, find area of $\triangle DEC$: area of $\triangle ABD$.

$ABCD$ 为一梯形，其中 $AB \parallel DC$ 及 $\triangle DCE$ 的面积 : $\triangle DCB$ 的面积 = 1 : 3。求 $\triangle DEC$ 的面积 : $\triangle ABD$ 的面积。



4. Let x be a positive integer. If $\frac{2}{3} \left(\frac{2}{3} \left(\frac{2}{3} (x-1) - 1 \right) - 1 \right)$ is divisible by 3, find the least possible value of x .

设 x 为一正整数。若 $\frac{2}{3} \left(\frac{2}{3} \left(\frac{2}{3} (x-1) - 1 \right) - 1 \right)$ 能被 3 整除，试求 x 之最小可能数值。

5. Pipe A alone takes 20 hours to fill a tank and pipe B takes 5 hours to fill the same tank alone. If pipes A and B together take x hours to fill the tank, find the value of x .

水管 A 能于 20 小时内独自盛满某一水池，而水管 B 则于 5 小时内完成此工作。若两水管同时使用时盛满这水池所需的时间则为 x 小时，求 x 。

6. Each interior angle of a regular polygon exceeds the exterior angle by 150° . Find the number of sides of the polygon.

一正多边形的每一内角被外角大 150° 。求此多边形的边的数目。

7. If $x + \frac{1}{x} = 3$, find the value of $x^2 + \frac{1}{x^2}$.

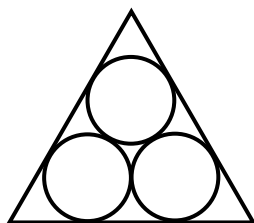
若 $x + \frac{1}{x} = 3$ ，求 $x^2 + \frac{1}{x^2}$ 的值。

8. Five numbers are in arithmetic progression. If the largest number is 7 times the smallest one and the average of the five numbers is 32, find the smallest number.

已知 5 个算术级数中之最大值为最小值之 7 倍，及该 5 个数之平均值为 32。求 5 个数中之最小值。

9. In the figure, three identical circles with radius r cm are tightly enclosed in a triangle. If the perimeter of the triangle is $(180 + 180\sqrt{3})$ cm, find r .

图中三个半径为 r cm 之全等圆被一三角形紧紧围着。若三角形之周界为 $(180 + 180\sqrt{3})$ cm，求 r 。



10. Two fair dice are thrown. Find the probability that the sum is less than 5 and at least one die is a '2'.

投掷两粒公平的骰子。求其总和为小于 5 及至少一粒骰子为 '2' 的机会率。